# Optimal mix of playground equipment

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# OPTIMAL MIX OF PLAYGROUND EQUIPMENT

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ABSTRACT

A two-stage procedure was developed to determine the optimal-mix of equipment (different types and numbers of each type) to be erected at playgrounds, with the objective to maximize the total enjoyment that children playing at the playground would derive from their use. At the first stage the contribution of each unit of equipment from each type to the total enjoyment of the children was determined using non-linear regression. The result of the first-stage was used as the input into a deterministic Dynamic Programming model. The solution procedure is demonstrated with an actual case example.



KEYWORDS
Dynamic Programming; Non-Linear Regression; Optimal Mix


## 1. INTRODUCTION

Park authorities and municipal recreation administrations must decide on the mix of playing equipment to be erected at public children playgrounds. It is clear that when the availability of space (area) for the playground does not pose any practical constraints, architectural, safety and environmental considerations are met, the more pieces of playing equipment are installed, and the wider their variety, the children using them will increase their satisfaction and enjoyment. However, all park authorities and recreational administrations are faced, without exceptions, with budgetary constraints. Thus, they have to decide on the optimal mix of the various playing equipment - what types of equipment and how many units from each type - while satisfying the budgetary constraints - that should be erected at playgrounds.

The problem of deciding on the optimal mix was addressed in various business and public circumstances. Dantzig [3] developed the Simplex method for the Linear Programming Model and produced examples pertaining to the optimal mix of products to be manufactured, as well as applications on optimal mix of crops to be produced. The range of fields in which optimal-mix problems were dealt with is enormous. Ladany [4], in the sports arena, determined the optimal mix of training activities to be practiced by a pentathlon athlete, and at the other end of the spectrum of problems, Ladany [5] dealt with the optimal mix of hotel room discriminatory pricing policies. The field of recreation management is abundant with thousands of research publications such as Smith & Headley [6] and Baron & Shechter [1], but none of them deals with the problem of determining the optimal mix of playground equipment.

Hence, the aim of this work is to determine the needed optimal mix of playground equipment for any budgetary constraint, and with the objective to maximize the total enjoyment that children using the playground derive from it. A general approach will be outlined, but it will be solved, using a real-life example from Israel, for neighborhood playgrounds in similar-sized localities attended by local children of similar socio-economic background. The approach consists of two steps: (1) the determination of the enjoyment children at playgrounds derive from each type of equipment using non-linear regression analysis, and (2) the derivation of the optimal equipment mix at the playground using deterministic Dynamic Programming which uses as input the results of the first step. The data are described in Section 2. The regression analysis is provided in Section 3, and the Dynamic Programming model and solution procedure is presented in Section 4. Concluding Remarks appear in the Summary.

## 2. THE DATA

A total of 25 neighborhood playgrounds were surveyed, having different mixes of playing equipment, and containing essentially only 4 different types of equipment: swings, slides, roundebouts (merry-go-rounds) and spring horses. The selected playgrounds had similar neighborhood sizes, and inhabitants with similar socio-economic characteristics, in which the peak use was on Saturdays between 10 A.M. and 2 P.M.

For each surveyed playground the number of the each type of the equipment available at that location was recorded. Likewise, the number of children playing or waiting for an engaged equipment was recorded. A fraction of the children were approached and each was requested to provide his/her score of satisfaction (enjoyment) from the equipment available at the playground on a scale from zero (lack of enjoyment) to 10 (full enjoyment). The collected data are presented in Table 1.

Since only a fraction of the children at the playground were polled, the total satisfaction of ALL the children at the playground was derived by multiplying the total satisfaction of the polled children at the given playground, by the inverse of the polling fraction. The resulting values are shown for each of the 25 playgrounds, in the bottom line of Table 1.

## 3. REGRESSION ANALYSIS

In order to evaluate the total enjoyment that one or more units of a given type of equipment (out of the mix of equipment) have on the children playing at the playground, it was assumed that each type of equipment generates an enjoyment which is independent of the availability and number of other equipment types at the same site. Hence, interactive influence of different equipment types, symbiosis effect - if it exists - was neglected. Therefore regression analysis was used.

The data for the regression is provided in Table 2, which lists for each of the 25 playgrounds of the sample:

$x_1$ - the number of roundebouts (merry-go-rounds),

$x_2$- the number of spring horses,

$x_3$- the number of slides, and

$x_4$- the number of swings.

Table 2 contains also, copied from the last line of Table 1,
(y - the total satisfaction of all the children at the playground).

Since increasing satisfactions were expected from additional units of the same equipment for some equipment types (because of the ill-effects of waiting), a non-linear regression was performed. Thus, as outlined in Bates & Watts [2], for each independent variable $X_i$, (i = 1, 2, …, 4), 3 additional variables were defined, having the forms $X_i^2$, $\frac{1}{X_i}$, and $\log X_i$

Thus y as the dependent variable was regressed against the [ 4 x (1+3) = 16 ] independent variables using a step-wise approach. The regression equation selected on the basis of high explained variation (R-square = 0.91), low standard error of estimate, significant regression coefficients, and economically logical behavior resulted in the following:

$$y = 16.2185 + 4.904X_1 + 4.787\left(\frac{1}{X_1}\right) + 7.128X_2 + 8.080X_3 + 9.698X_4 \qquad (1)$$

showing non-linear influence of $X_1$ , and linear influence of $X_2$, $X_3$ & $X_4$.

## 4. DYNAMIC PROGRAMMING

Assume that each unit of equipment of type n costs $C_n$ and reduces the remaining budget available for units of equipment of other types by this amount. If we define the total satisfaction derived ONLY from the $X_n$ units of equipment of type n erected at the playground as $V_n(X_n)$ , and also define the total satisfaction derived from the playground which contains $X_n$ units of equipment of type n, and erected with a total equipment budget S (accommodating the possible erection of units also from additional equipment types from the same budget) as $F_n(S,X_n)$, where the total budget is given in Dollars or other monetary units, it is possible to express a recursive relationship based on the principle of optimality (for example see Taha [7] ):

$$F_n(S,X_n) = V_n(X_n) + F^*_{(n+1)}(S - C_nX_n, X_{(n+1)}) \quad , \qquad (2)$$

where

$$F^*_n = \underset{X_n}{MAX}\, F_n(S,X_n) \quad . \qquad (3)$$

Such a Dynamic Programming Formulation calls for a backward solution procedure.

The model is general, and we will solve it for the present numerical case. From equation (1) it is possible to derive a table of $V_n(X_n)$ , for all the (Max n = 4) possible equipment types. It is shown in Table 3.

According to the local park administrators, the budget available for the erecting of equipment at playgrounds (while maintenance is available from other budgets) is S = 8,000 monetary units. Likewise, in interviews with a major supplier of playground equipment it was found that the unit cost of equipment is as shown below in Table 4.

The sequenced backward solution is presented in Tables 5 to 8.

The Optimal Solution is obtained in reverse order from Tables 5 to 8, starting with Table 5:

$X_1^* = 1$, from $F_1^*(8{,}000, X1)$

$X_2^* = 1$, from $F_2^*(8{,}000 - 600X_1^*, X_2)$ □ $(8{,}000 - 600X_1^*) = 7{,}400$

$X_3^* = 0$, from $F_3^*(7{,}400 - 1{,}200X_2^*, X_3)$ □ $(7{,}400 - 1{,}200X_2^*) = 6{,}200$

$X_4^* = 4$, from $F_4^*(6{,}200 - 1{,}500X_3^*, X_4)$ □ $(6{,}200 - 1{,}500X_3^*) = 6{,}200$

## 5. CONCLUDING REMARKS

Although the approach outlined here for the derivation of the optimal equipment mix at playgrounds is solved for a specific case, the two-stage procedure is general. The first stage - the determination of the contribution of each unit of equipment of a given type to children's enjoyment at playground - might vary from case to case, but it is only the input to the second and main stage of the general Dynamic Programming optimal-mix model.

While the suggested approach is not a panacea, it is an important first step to cope with the problem of deriving in a scientific way the optimal mix of playground equipment, instead of basing the decision only on intuition. The impact of additional factors (variables), like the number of potential users of the playground and the variation in their socio-economic characteristics, more equipment types, etc. can be accommodated by deriving more sophisticated first-stage input functions. Likewise, differential area requirements for each type of equipment, differential maintenance costs, constraint on the total area for the playground, etc. could be accommodated by extending the Dynamic Programming formulation, while retaining the same recursive nature of the relationship.

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TABLE 1 : Part 1 . The Total Satisfaction Score for each Child at each Playground.

| | Playground Number | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Child** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 1 | 8 | 7 | 9 | 9 | 10 | 8 | 9 | 6 | 9 | 7 | 8 | 10 | 8 |
| 2 | 7 | 6 | 7 | 7 | 9 | 8 | 8 | 4 | 9 | 7 | 8 | 6 | 7 |
| 3 | 6 | 8 | 8 | 8 | 10 | 7 | 7 | 7 | 8 | 9 | 7 | 8 | 7 |
| 4 | 9 | 7 | 8 | 8 | 8 | 7 | 8 | 6 | 7 | 8 | 9 | 7 | 6 |
| 5 | 10 | 6 | 9 | 7 | 7 | 7 | 8 | 5 | 7 | 8 | 8 | 8 | 8 |
| 6 | 8 | 7 | 7 | 8 | 9 | 8 | 7 | 8 | 8 | 7 | 7 | 7 | 7 |
| 7 | 7 | | 6 | 7 | 8 | 9 | 9 | | 8 | 8 | 7 | 8 | |
| 8 | | | 8 | | 10 | 8 | 8 | | 9 | 8 | 5 | 7 | |
| 9 | | | 10 | | 9 | 7 | 6 | | 7 | 7 | | | |
| 10 | | | | | 8 | | 7 | | 9 | | | | |
| 11 | | | | | 8 | | | | 7 | | | | |
| 12 | | | | | 9 | | | | 10 | | | | |
| Number of children *polled* | 7 | 6 | 9 | 7 | 12 | 9 | 10 | 6 | 12 | 9 | 8 | 8 | 6 |
| Total Satisfaction of the children *polled* at the playground | 55 | 41 | 72 | 54 | 105 | 69 | 77 | 36 | 98 | 69 | 59 | 61 | 43 |
| Total Number of the children at the playground | 15 | 9 | 18 | 15 | 24 | 18 | 23 | 12 | 25 | 19 | 16 | 17 | 12 |
| Total Satisfaction of the children at the playground | 117.86 | 61.50 | 144.00 | 115.71 | 210.00 | 138.00 | 161.00 | 72.00 | 204.17 | 145.67 | 118.00 | 129.63 | 86.00 |

TABLE 1 : Part 2 . The Total Satisfaction Score for each Child at each Playground.

| | Playground Number | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Child** | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| 1 | 7 | 7 | 8 | 7 | 7 | 5 | 7 | 7 | 8 | 8 | 8 | 7 |
| 2 | 9 | 6 | 7 | 6 | 7 | 5 | 8 | 6 | 9 | 8 | 9 | 8 |
| 3 | 8 | 6 | 7 | 5 | 8 | 7 | 7 | 6 | 7 | 9 | 7 | 8 |
| 4 | 7 | 4 | 8 | 7 | 6 | 6 | 6 | 5 | 7 | 7 | 9 | 6 |
| 5 | 7 | 7 | 5 | 7 | 6 | 7 | 9 | 7 | 8 | 8 | 9 | 7 |
| 6 | 10 | 7 | 9 | | 7 | 4 | 8 | 7 | 7 | 7 | 7 | 6 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 6 | 8 | | 7 | | | | 6 | 7 | 7 | |
| 8 | | | 6 | | | | | | | 8 | 8 | |
| 9 | | | | | | | | | | | 8 | |
| 10 | | | | | | | | | | | 7 | |
| 11 | | | | | | | | | | | | |
| 12 | | | | | | | | | | | | |
| Number of children *polled* | 7 | 7 | 8 | 5 | 7 | 6 | 6 | 6 | 7 | 8 | 10 | 6 |
| Total Satisfaction of the children *polled* at the playground | 56 | 43 | 58 | 32 | 48 | 34 | 45 | 38 | 52 | 62 | 79 | 42 |
| Total Number of the children at the playground | 14 | 7 | 16 | 10 | 14 | 6 | 13 | 8 | 15 | 16 | 20 | 11 |
| Total Satisfaction of the children at the playground | 112.00 | 43.00 | 116.00 | 64.00 | 96.00 | 34.00 | 97.50 | 50.67 | 111.43 | 124.00 | 158.00 | 77.00 |

TABLE 2. Number of Pieces of Equipment of Each Type at the Playgrounds Surveyed

| **Playground Number** | **Number of Roundabouts at Playground** <br> **$X_1$** | **Number of Spring Horses at Playground** <br> **$X_2$** | **Number of Slides at Playground** <br> **$X_3$** | **Number of Swings at Playground** <br> **$X_4$** | **Total Satisfaction of all Children from the Playground** <br> **y*** |
|---|---|---|---|---|---|
| 1 | 3 | 3 | 3 | 4 | 117.85 |
| 2 | 1 | 2 | 2 | 2 | 61.50 |
| 3 | 4 | 5 | 4 | 4 | 144.00 |
| 4 | 3 | 3 | 3 | 3 | 115.71 |
| 5 | 8 | 8 | 3 | 7 | 210.00 |
| 6 | 3 | 4 | 6 | 3 | 138.00 |
| 7 | 7 | 6 | 2 | 6 | 161.00 |
| 8 | 0 | 2 | 4 | 4 | 72.00 |
| 9 | 8 | 8 | 3 | 8 | 204.16 |
| 10 | 3 | 4 | 6 | 3 | 145.66 |
| 11 | 3 | 5 | 2 | 3 | 118.00 |
| 12 | 6 | 4 | 3 | 2 | 129.63 |
| 13 | 0 | 3 | 1 | 2 | 86.00 |
| 14 | 3 | 4 | 2 | 3 | 112.00 |
| 15 | 2 | 0 | 1 | 0 | 43.00 |
| 16 | 5 | 6 | 3 | 0 | 116.00 |
| 17 | 3 | 3 | 2 | 2 | 64.00 |

| | | | | | |
|---|---|---|---|---|---|
| 18 | 4 | 5 | 1 | 2 | 96.00 |
| 19 | 0 | 3 | 0 | 1 | 34.00 |
| 20 | 5 | 4 | 2 | 2 | 97.50 |
| 21 | 3 | 1 | 2 | 0 | 50.66 |
| 22 | 4 | 6 | 3 | 2 | 111.42 |
| 23 | 0 | 3 | 2 | 4 | 124.00 |
| 24 | 6 | 4 | 3 | 4 | 158.00 |
| 25 | 3 | 3 | 2 | 1 | 77.00 |

Table 3. Values of $V_n(X_n)$ for $X_n$ Multiples of Equipment Type n (Derived from Equation (1) )

| | **n** | | | |
|---|---|---|---|---|
| $X_n$ | **1 (Roundebout)** | **2 (Spring Horse)** | **3 (Slide)** | **4 (Swing)** |
| 1 | 9.691 | 7.128 | 8.080 | 9.698 |
| 2 | 12.202 | 14.256 | 16.160 | 19.396 |
| 3 | 16.307 | 21.384 | 24.240 | 29.094 |
| 4 | 20.810 | 28.512 | 32.320 | 38.792 |

Table 4. Cost per Unit of Equipment Type n -$C_n$

| **n : Type of Equipment** | **$C_n$ : Cost per Unit for Equipment Type n** |
|---|---|
| 1 Roundebouts | 600 |
| 2 Spring Horses | 1,200 |
| 3 Slides | 1,500 |
| 4 Swings | 1,500 |

Table 5. Values of $F_4(S,X_4) = V_4(X_4)$ for Various Values of S and the Resulting Optimal Values of $F^*_4$ $(S,X_4)$ and $X^*_4$

| | **$X_4$** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **S** | **0** | **1** | **2** | **3** | **4** | **5** | **$F^*_4$ $(S,X_4)$** | **$X^*_4$** |
| **0** | 0 | | | | | | 0 | 0 |
| **1,500** | 0 | 9.698 | | | | | 9.698 | 1 |
| **3,000** | 0 | 9.698 | 19.396 | | | | 19.396 | 2 |
| **4,500** | 0 | 9.698 | 19.396 | 29.094 | | | 29.094 | 3 |
| **6,000** | 0 | 9.698 | 19.396 | 29.094 | 38.792 | | 38.792 | 4 |
| **7,500** | 0 | 9.698 | 19.396 | 29.094 | 38.792 | 48.490 | 48.490 | 5 |

Table 6. Values of $F_3(S,X_3) = V_3(X_3) + F^*_4$ $(S-C_3X_3,X_4)$ for Various Values of S and the Resulting Optimal Values of $F^*_3$ $(S,X_3)$ and $X^*_3$

| | **$X_3$** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **S** | **0** | **1** | **2** | **3** | **4** | **5** | **$F^*_3$ $(S,X_3)$** | **$X^*_3$** |
| **0** | 0 | | | | | | 0 | 0 |
| **1,500** | 9.698 | 8.080 | | | | | 9.698 | 0 |
| **3,000** | 19.396 | 17.778 | 16.160 | | | | 19.396 | 0 |
| **4,500** | 29.094 | 27.476 | 25.858 | 24.240 | | | 29.094 | 0 |
| **6,000** | 38.792 | 37.174 | 35.556 | 33.938 | 32.320 | | 38.792 | 0 |
| **7,500** | 48.490 | 46.872 | 45.254 | 43.636 | 42.018 | 40.400 | 48.490 | 0 |

Table 7. Values of $F_2(S,X_2) = V_2(X_2) + F^*_3$ $(S-C_2X_2 , X_3)$ for Various Values of S and the Resulting Optimal Values of $F^*_2$ $(S,X_2)$ and $X^*_2$

| S | $X_2$ | | | | | $F^*_2$ (S,$X_2$) | $X^*_2$ |
|---|---|---|---|---|---|---|---|
| | **0** | **1** | **2** | **3** | **4** | | |
| **0** | 0 | | | | | 0 | 0 |
| **1,200** | 0 | 7.128 | | | | 7.128 | 1 |
| **1,500** | 9.698 | 7.128 | | | | 9.698 | 0 |
| **2,400** | 9.698 | 7.128 | 14.256 | | | 14.256 | 2 |
| **3,000** | 19.396 | 16.826 | 14.256 | | | 19.396 | 0 |
| **3,600** | 19.396 | 16.826 | 14.256 | 21.384 | | 21.384 | 3 |
| **4,500** | 29.094 | 22.524 | 23.954 | 21.384 | | 29.094 | 0 |
| **4,800** | 29.094 | 22.524 | 23.954 | 21.384 | 28.512 | 29.094 | 0 |
| **6,000** | 38.792 | 36.222 | 33.652 | 31.082 | 28.512 | 38.792 | 0 |
| **7,200** | 38.792 | 45.920 | 43.350 | 40.780 | 38.210 | 45.920 | 1 |
| **7,500** | 48.490 | 45.920 | 43.350 | 40.780 | 38.210 | 48.490 | 0 |

Table 8. Values of $F_1(S,X_1) = V_1(X_1) + F^*_2$ $(S\text{-}C_1X_1,X_2)$ and the Resulting Optimal Values of $F^*_1$ $(S,X_1)$ and $X^*_1$, for the Total Budget of S = 8,000

| S | $X_1$ | | | | | $F^*_1$ (8,000 , $X_1$) | $X^*_1$ |
|---|---|---|---|---|---|---|---|
| | **0** | **1** | **2** | **3** | **4** | | |
| **8,000** | 48.490 | 55.611 | 50.994 | 50.009 | 49.904 | 55.611 | 1 |